Logaritma

$x = b^{y}$ → exponent

base: b

$b, x > 0$ and $b \neq 1$

$\log_b(x) = y$ → exponent

base: b

Log Özellikleri

1) $\log_a x + \log_a y = \log_a(xy)$ ✓

2) $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$ ✓

3) $\log_a x^n = n \cdot \log_a x$ //

4) $\log_a 1 = 0$ $\quad a^0 = 1$

5) $\log_a a = 1$ $\quad a^1 = a$

6) $\log_a\left(\frac{1}{x}\right) = \log_a(x^{-1}) = -\log_a x$

7) $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log_{10} x}{\log_{10} a} = \log_a x$ //

8) $\log_a x = \frac{1}{\log_x a}$ //

[Maximum mark: 7]

Find the value of each of the following, giving your answer as an integer.

(a) $\log_{10} 100$. $\Rightarrow 10^x = 100 \quad x = 10$

(b) $\log_{10} 50 + \log_{10} 2$. $\Rightarrow \log_{10} 100 = 10$

(c) $\log_{10} 4 - \log_{10} 40$. $\Rightarrow \log_{10}\left(\frac{4}{40}\right) = \log_{10}\left(\frac{1}{10}\right) = -1$

$\log_{10} x = \log x$

[Maximum mark: 7]

$\log_e 2$ $\quad \ln 6 = \log_e 6$

Let $p = \ln 2$ and $q = \ln 6$. Write down the following expressions in terms of p and q.

(a) $\ln 12$

(b) $\ln 3$

(c) $\ln 48$

$2^2 \times 6$

a) $\ln 2 + \ln 6$
$= p + q$

b) $\ln 6 - \ln 2$
$= \ln 3$
$= p - q$

c) $\ln 2 + \ln 2 + \ln 6$
$= \ln 48$
$= 2p + q$

[Maximum mark: 6]

Let $\log_2 a = p$, $\log_2 b = q$, $\log_2 c = r$. Write down the following expressions in terms of p, q and r.

(a) $\log_2\left(\frac{ab}{c}\right)$

(b) $\log_2\left(\frac{a^2c}{b^3}\right)$

(c) $\log_a b$ $= \frac{\log_x b}{\log_x a} = \frac{\log_2 b}{\log_2 a}$

a) $(\log_2 a + \log_2 b) - \log_2 c$

$= p + q - r$

b) $2p + r - 3q$

c) $\frac{q}{p}$

[Maximum mark: 5]

Solve the equation $4\log_5\sqrt{x} - \frac{1}{\log_3 5} = 3\log_5(3x^2)$, where $x > 0$.

$-\log_5 3$

$x^{\frac{1}{2}\cdot 4} = x^2$

$4\log_5\sqrt{x} - \log_5 3 - 3\log_5(3x^2) = 0$

$\Rightarrow \log_5(\sqrt{x})^4 - \log_5 3 - \log_5(3x^2)^3 = 0$

$\Rightarrow \log_5 x^2 - \log_5 3 - \log_5 27x^6 = 0$

$\log_5\left(\frac{x^2}{3.27x^6}\right) = 0$

$\frac{x^2}{3.27x^6} = 1$

$x^2 = 3.27x^6$

$x^4 = 81x^6$

$x^{-4} = 81$

$x = \frac{1}{3}$

[Maximum mark: 5]

Solve the equation $15^{4a} = 81^{a+2}$ for a. Express your answer in terms of $\ln 3$ and $\ln 5$.

$$3^{4a} \cdot 5^{4a} = 3^{4a} \cdot 3^{8}$$

$$15^{4a} = 3^{4a} \cdot 5^{4a}$$

$$81^{a+2} = 3^{4(a+2)} = 3^{4a+8}$$

$$3^{4a} \cdot 5^{4a} = 3^{4a+8}$$

$$3^{4a} \cdot 5^{4a} = 3^{4a} \cdot 3^{8}$$

$$5^{4a} = 3^{8}$$

$$\ln 5^{4a} = \ln 3^{8}$$

$$4a \cdot \ln 5 = 8 \cdot \ln 3$$

$$a \cdot \ln 5 = 2 \cdot \ln 3$$

$$a = \frac{2 \cdot \ln 3}{\ln 5}$$